CHISA'96

12th International Congress
of Chemical and Process Engineering
25–30 August 1996, Praha, Czech Republic
Scientific Committee

Mail: Congress CHISA, P. O. Box 857, 111 21 Praha 1 Fax: +42 2 342073 E-mail: chisa@icpf.cas.cz

8 July, 1996

L. M. Uliev
Kharkov State Polytech. Univ.
ul. Frunze 21
310002 Kharkov
Ukraine

Dear Participant,

I am pleased to inform you that your paper has been included in the Final Program of the Congress as follows:

Paper No.: P7.115 Ref. No.: 0273
Author: Uliev, L. M.
Affiliation: State Polytech. Univ., Kharkov, Ukraine
Title: Pressure drop-flowrate characteristic for non-isothermal flow high-viscosity liquids in circular channel

Presentation: poster, Thursday

I am looking forward to meeting you in Prague.

Sincerely yours,

Ivan Wichterle
Scientific Committee

PRESSURE DROP- FLOWRATE CHARACTERISTIC FOR NON- ISO-THERMAL FLOW HIGH-VISCOSITY LIQUIDS IN CIRCULAR CHANNEL

L.M. Uliev
Kharkov State Polytech. Univ., Dep. Chem. Eng., Frunze St. 21, 310002, Kharkov, Ukraine;
Tel. (380) 572-400-893, Fax. (380) 572-400-632, E-mail: ulm@lotus.kpi.kharkov.ua

Results of numerical investigations of pressure-flowrate characteristic for flow of high-viscosity liquids with heat exchange with ambient and taking into account dissipation are represented here. Intensity heat exchange is defined by temperature of ambient and number Bi. Pressure flow-rate characteristic is unmonotonous. The influence of dependence of $\Delta P(V_0)$ on stability of work of die arrangement of extrusion machines is discussing.

For production and processing of thermoplastic polymer materials their melts are pressed through thin forming channels- the dies. They have cylindrical form for the production of the strands, bars and the pellets. As melts of polymers are high-viscosity liquids, so the flow in the dies occurs in condition of the large gradients of temperatures and viscosity in consequence of considerable dissipation of energy and of sharp dependence of viscosity from the temperature. The distribution of viscosity in die is determined by pressure-flowrate characteristic of the die, from which we may choose the optimum technological and structural parameters of process of extrusion [1].

Pressure-stream characteristic with non-isothermal flow of viscosity liquids was investigated earlier principally for the special case of flow on the region of established heat transfer [2,3] or for flow with negligible dissipation [4-6].

The boundary conditions there had been chosen as a rule either with given temperature of the wall [2,4-6] or of the second kind [4]. Works [7-10] had dealt with the characteristics for the boundary condition of the third kind for the thermally non-developed flow with constant temperature in cross section that allowed authors take into account the axial convection heat transfer only in mean but the across convection heat transfer was not taken into consideration. Non-monotonous pressure-stream curve was obtained in [4,10] but in [10] only for the flat case. Non-isothermal dependence $\Delta P(V_0)$ for oil flow in a circular capillary with the thermal boundary condition of the first kind and for flow with the adiabatic wall was investigated in [11]. Here dissipation of energy and across convection transfer are taken into account. Received dependence $\Delta P(V_0)$ is strictly monotonous, but the numbers Nahme-Griffith here are on 3-4 order less than for flow of melt polymers with the same velocity.

This work investigates pressure-flowrate dependence for the flow of thermoplastic polyurethane (TPU) in the circular cylindrical channels with different intensity heat exchange with ambient medium.

Melts of glue kinds TPU (Vitur T-12K and et al.) within alteration of parameters of processing are alike Newtonian liquids with temperature dependence of viscosity [12]

$$\mu(T)=\mu_0 \exp\left[\frac{E}{R}\left(\frac{1}{T}-\frac{1}{T_0}\right)\right], \qquad (1)$$

where $\mu_0 = 10^3$ Pa·s, $E \sim (10^5 \ldots 3\cdot 10^5)$ J/mol, $T_0 = 463°$.

For expenditure presenting practical interest $W = 10^{-5} \ldots 10^{-7}$ $m^3 s^{-1}$, physical of property $\mu_0 = 10^3$ Pa·s, $\rho \sim 1200$ kg m^{-3}, $\alpha \sim 10^{-7} m^2 s^{-1}$ and $r_0 \sim (1\ldots 3)\cdot 10^{-3}$m, number $Re << 10^{-2}$, $Pe \sim 10^5$, and then lengths of mechanical l_1 and of thermal l_2 relaxation form quantities [13]

$$l_1 \approx \frac{V_0 r_0^2 \rho}{\mu} \approx 10^{-6} m, \quad l_2 \approx \frac{V_0 r_0^2 \rho}{\mu} \approx 10^{-6} m, \qquad (2)$$

i.e. on the entry in channel we can account profile of velocity fully developed and corresponding to distribution of temperature. If liquid enters the channel with uniform distribution of temperature then the main change in distribution of velocity takes place on the length of small viscosity forming the shear layer [13]

$$l_3 \approx Pe \cdot Gn^{-3/2} r_0 \approx 10^{-1} m. \qquad (3)$$

The taken data, values l_1, l_2, l_3, numbers Re, Pe, and axial symmetry of problem allow to estimate values of derivatives in equation of the convection heat transfer, to receive the correlation between radial and longitudinal components of velocity $V_r = {}^{r_0}/_{L_0} = o(V_z)$ (here $L_0 = l_3$ for flow with uniform initial profile of temperature, l_2- for flow with preliminary formed small viscosity layer [14,15]), that allow us to simplify the steady-state set of equations of hydrodynamics and heat exchange [1] which employing dimensionless variables and parameters

$$\xi=\frac{r}{r_0},\ \chi=\frac{z}{r_0},\ \Pi=\frac{(P-P_0)r_0}{\mu_0 V_0},\ V_0=\frac{W}{\pi r_0^2},\ \Theta=\frac{T-T_0}{\Delta T},\ v=\frac{V_z}{V_0},\ \omega=\frac{V_r}{V_0},\ m=\frac{\mu}{\mu_0}=\exp\left(-\frac{\Theta}{1+\beta\Theta}\right),$$

$$Gn=\frac{\mu(T_0)V_0^2}{\lambda \Delta T},\ Bi=\frac{K r_0}{\lambda},\ Pe=\frac{V_0 r_0}{a},\ \Delta T=\left|\mu(T)\Big/\frac{\partial \mu}{\partial T}\right|_{T=T_0}=\beta T_0,\ \beta=\frac{RT_0}{E},$$

write down in shape

$$\frac{\partial \Pi}{\partial \chi}=\frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\xi m \frac{\partial v}{\partial \xi}\right), \qquad 0\le\xi<1,\ 0\le\chi\le L/_{r_0}, \qquad (4)$$

$$\frac{1}{\xi}\frac{\partial}{\partial \xi}(\xi\omega)+\frac{\partial v}{\partial \chi}=0, \qquad (5)$$

$$\frac{1}{\xi}\frac{\partial}{\partial \xi}(\xi\omega\Theta)+\frac{\partial}{\partial \chi}(v\Theta)=\frac{1}{Pe}\frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\xi\frac{\partial \Theta}{\partial \xi}\right)+\frac{Gn}{Pe}m\left(\frac{\partial v}{\partial \xi}\right)^2 \qquad (6)$$

Boundary conditions will be written

$$\frac{\partial \Theta}{\partial \xi}=0,\ \frac{\partial v}{\partial \xi}=0, \qquad \xi=0,\ 0\le\chi\le L/_{r_0}, \qquad (7)$$

$$v=0,\quad \omega=0,\ \frac{\partial\Theta}{\partial\xi}=-Bi(\Theta-\Theta_a),\qquad \xi=1,\ 0\le\chi\le L/r_0, \tag{8}$$

$$\Theta=0,\quad \Pi=0,\qquad 0\le\xi\le 1,\quad \chi=0, \tag{9}$$

but condition of constant of flow rate will be written

$$2\int_0^1 v\xi d\xi=1. \tag{10}$$

For solving this problem the region of flow is separated into N concentric cylindrical layers, and using the method elaborated early [14,15] ,we receive the set of ordinary differential equations describing the mean temperatures in the layers and pressure

$$\frac{d\overline{\Theta}_i}{d\chi}=\frac{2}{\overline{v}_i d_i}\left[\xi_i(\omega_i-St_i)(\overline{\Theta}_i-\overline{\Theta}_{i+1})-(\omega_{i-1}+St_{i-1})(\overline{\Theta}_i-\overline{\Theta}_{i-1})\right]+\frac{Gn\cdot m_i}{Pe\cdot\overline{v}_i}\overline{\left(\frac{\partial v_i(\xi)}{\partial\xi}\right)^2}, \tag{11}$$

$$\frac{d\Pi}{d\chi}=-8\left(\sum_{i=1}^{N}\frac{\xi_i^4-\xi_{i-1}^4}{m_i}\right)^{-1}=-\frac{8}{S}, \tag{12}$$

where $\overline{v}_i=\frac{1}{S}\left(\frac{d_i}{m_i}+2\sum_{K=1}^{N-i}\frac{d_{i+K}}{m_{i+K}}\right)$, $\omega_i=-\frac{1}{2\xi_i}\sum_{K=1}^{i}d_K\frac{d\overline{v}_K}{d\chi}$, $d_i=\xi_i^2-\xi_{i-1}^2$,

$St_1=\frac{1}{\xi_1}\left(\frac{\xi_2}{d_2}\ln\frac{\xi_2}{\xi_1}-\frac{1}{2}\right)^{-1}$, $St_i=\frac{1}{\xi_i}\left(\frac{\xi_{i+1}}{d_{i+1}}\ln\frac{\xi_{i+1}}{\xi_i}-\frac{\xi_{i-1}}{d_i}\ln\frac{\xi_i}{\xi_{i-1}}\right)^{-1}$, $St_N=\left(\frac{1}{2}-\frac{\xi_{N-1}^2}{d_N}\ln\frac{\xi_N}{\xi_{N-1}}+\frac{1}{N_{Bi}}\right)^{-1}$,

where i= 1,2,…,N and then $\overline{\Theta}_{N+1}=\Theta_a$.

Derivative $\frac{d\overline{v}_i}{d\chi}$ and $\frac{dv_i}{d\xi}$ determine from distribution $v_i(\xi)$

$v_i(\xi)=\frac{2}{S}\left(\frac{\xi_i^2-\xi^2}{m_i}+\sum_{K=1}^{N-i}\frac{d_{i+K}}{m_{i+K}}\right)$, and expression $\frac{dm_i}{d\chi}=-\frac{m_i}{1+\beta\overline{\Theta}_i}\frac{d\overline{\Theta}_i}{d\chi}$.

First let us consider, the dependence of $\Delta P(V_0)$ on the part of channel with the adiabatic wall and with length L=0.12 m and $r_0=1.5\cdot10^{-3}$ m for parameters $\alpha=8\cdot10^{-8} m^2s^{-1}$, $\beta=1.44\cdot10^{-2}$ (Figure 1c).

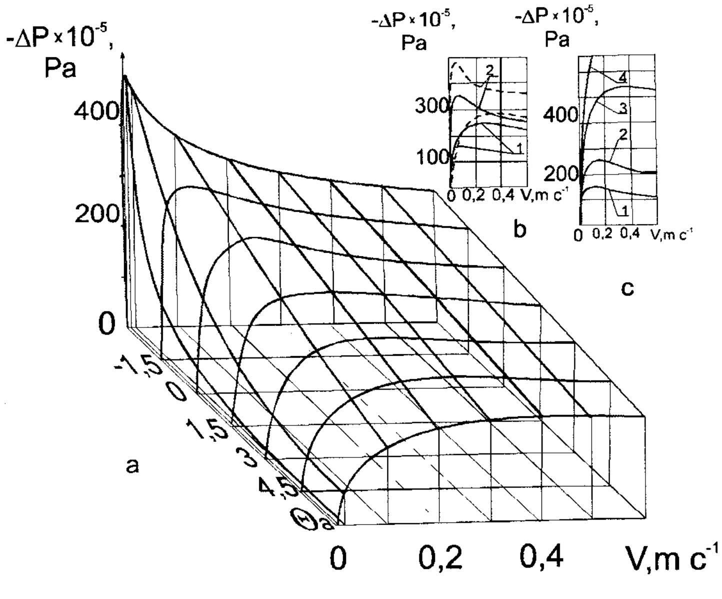

Figure 1. Pressure drop in cylindrical channel a) *vs.* average of velocity of flow and ambient temperature for Bi = 3.75; b) solid lines for flow with Bi = 3.75, dashed for Bi = 15; 1- for $\Theta_a=3$, **2-**$\Theta_a=-1.5$; c) flow for adiabatic wall: 1-β = 0.77·10^{-3}, 2- β = 1.44·10^{-2}, 3-3.88·10^{-2}, 4-7.7·10^{-2}.

For a little expenditure of liquid shear rate is small, dissipation is insignificant and gen-

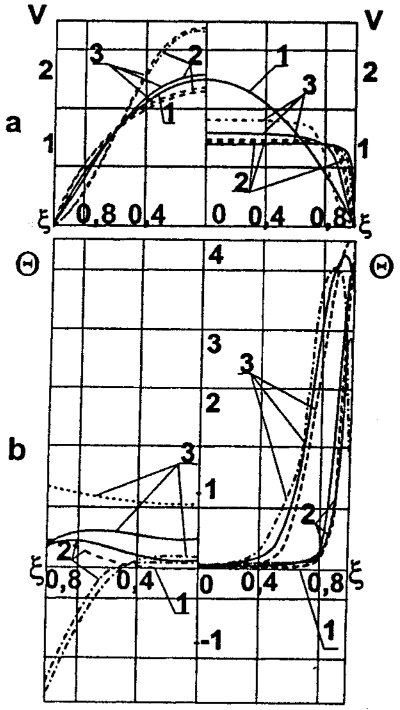

Figure 2. Distribution a) dimentionless longitudinal velocity. -b) dimentionless temperature across channel. Left half for small velocity V_0~0.01 m/s, right half for V_0~0.1...0.3 m/s. Solid lines - $\Theta_a = 0$, dash-dotted lines $\Theta_a=-3$, dashed lines for Bi = 0. 1- for $\chi = 0$, 2- $\chi = 7.8$, 3 - $\chi = 80$.

erating heat has time to distribute uniformly on cross-section of stream. Although temperature of liquid in total increases slightly along the channel, the profile of velocity differs a little from Poiseuille profile (Figure 2), that leads only to small deviation dependence $\Delta P(z/r_0)$ from linear (Figure 3a). With increasing of flow rate this behaviour will be until the boundary layer with little viscosity will appear. With rise of temperature at the wall viscosity is decreasing there, profile of velocity is becoming more and more filled, *i.e.* increasing shear rate on the periphery carries out to localisation of heat generate. At the same time module of gradient of pressure is decreasing and curve $\Delta P(\chi)$ with appearance of small-viscosity layer appreciably deviates from linear (Figure 3a).

For some more expenses of melt, considerable dissipation leads to fast arise of high-temperature small-viscosity layer and to almost plug-flow (Figure 2), but it only intensifies the described effects with further flow. In consequence of it the high-temperature plug-flow with little gradient of pressure is forming in channel.

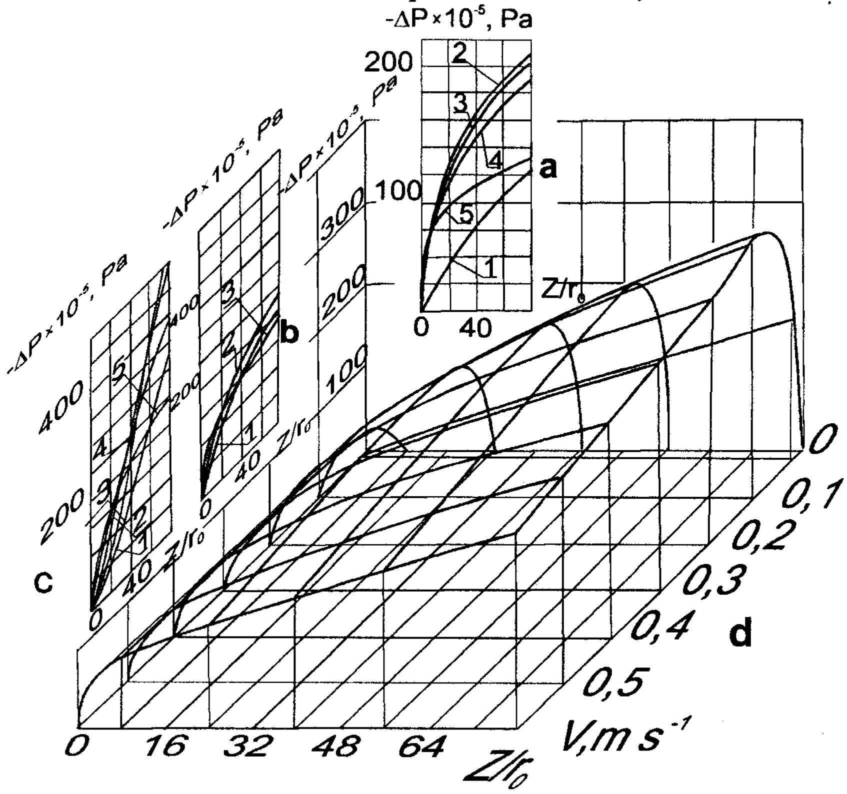

Figure 3. Distribution of pressure along channel a) flow with adiabatic wall, 1- $V_0= 1.44\cdot10^{-2}$ m/s, 2 - $V_0 = 1.44\cdot10^{-1}$ m/s, 3-2.82 $\cdot10^{-1}$ m/s; b) flow for $\Theta_a = 3$, 1- $V_0 = 8.48\cdot10^{-2}$ m/s, 2-V_0~ (1.4...4) $\cdot10^{-1}$ m/s, 3- 1.13 m/s; c) flow for $\Theta_a = -3$, 1- $V_0= 1.13 \cdot10^{-3}$ m/s, 2- 4.24 $\cdot10^{-3}$ m/s, 3 - $1.41\cdot10^{-2}$ m/s, 4- 8.48 $\cdot10^{-2}$ m/s, 5 - 0.71 m/s m/s; d) pressure *vs.* Longitudinal co-ordinate and mean velocity for Θ_a=0.

As a result pressure drop in channel may be less than one for flow with small expense melt, although decrease of pressure on entrance in first case is big (Figure 3a). In result pressure-stream characteristic is non-monotonous with non-isothermal flow.

In case of heat exchange with ambient medium non-isothermal flow is not defining only by dissipation, but by heat transfer on the boundary too. For $\Theta_a=0$ and Bi =3.75 more energy is spent on forming of the small viscosity layer because of heat exchange with ambient (Figure 2) and so condition $|\Delta P(\Theta_a=0)|>|\Delta P(Bi=0)|$ is performing here.

It will be noted that for short channels $L < 15r_0$ pressure-stream function monotonously increases (Figure 3d). For big V_0 namely here the forming of small-viscosity shear layer occur and with increasing flow rate, $|\nabla P|$ only increases.

For small flow rate and $\Theta_a>0$ with Bi =3.75 liquid is heated uniformly (Figure 2) therefore pressure falls slower than in case described earlier (Figure 3b). For big V_0 liquid has no residence for uniform heat, but here small-viscosity layer appears at boundary, on generation of which it does not spend energy, and then correlation $|\Delta P(\Theta_a>0)|<|\Delta P(\Theta_a=0)|\forall V_0$ is fulfilling (Figure 1a).

For case $\Theta_a<0$ and small velocity, viscosity of melt is decreasing on the periphery, profile of velocity is striking, dissipation energy is becoming considerable in central field, too. Therefore, liquid is cooled on section non-uniform (Figure 2) but central part is even heated a little, but $|\Delta P|$ along the channel is increasing because of increasing μ on the periphery (Figure 3c) (convexity curve $\Delta P(\chi)$ is down). For the big V_0 temperature near wall is significantly smaller than for other Θ_a (Figure 2), therefore the condition $|\Delta P(\Theta_a<0)|>|\Delta P(\Theta_a=0)|\forall V_0$ fulfils here (Figure 1a).

It should be noted that increase of V_0 with $\Theta_a<0$ power dissipation very quickly becomes predominate over heart exchange with environment therefore maximum curve $|\Delta P(V_0)|$ displaces to the left. Notice, too, that for little velocity dependence $\Delta P(V_0)$ mainly is alike function $\mu=\mu(T)$ as dissipation effects here are inconsiderable, but for high-temperature of the flow $\Delta P(\Theta_a)$ is becoming practically linear (Figure 1).

Intensity heat exchange with environment for constants Θ_a is defining by value Bi. Increase of Bi for $\Theta_a>0$ carries out to that for small V_0 liquid is heating more intensively and $|\Delta P|$ is decreasing. For big velocity the temperature is becoming bigger Θ_a in consequence of dissipation and flux heat $q=Bi(\overline{\Theta}_N-\overline{\Theta}_a)$ changes sign and his absolute value is more than for smaller Bi that carries out to increasing $|\Delta P(V_0)|$ (Figure 1b). It is clear that at $\Theta_a<0$ increasing of Bi will carry out only to rising $|\Delta P|$ for any V_0 (Figure 1b).

The value β exerts influence on the dependence $|\Delta P(V_0)|$. For $\beta\to 0$, $Gn\to 0$, *i. e.* intensity of dissipation does not influence on the dynamic flow. Number Gn and influence of the temperature on μ are rising with decrease of β therefore value of reached maximum pressure is decreasing and this pressure is reached on smaller velocities (Figure 1c).

Described effects can bring to unexpected phenomena in processing of polymers. For instance, if die plate is used without heat for under granulation as a rule, several tens of holes become to be obstructed by thicken melt and it is pressing through remain 1...3 holes with previous flow rate and without change the pressure in the extruder.

This situation can arise in consequence of some, possibly of casual, decreasing of velocity of flow in some dies that carries out to increasing of residence of melt there. It is cooled more than in other dies so its velocity is decreasing and so on, until the flow stops. At the same time in other dies velocity accordingly increases, rising dissipation of energy, too, but $|\nabla P|$ decreases, *etc.* After all the whole volume of melt will be pressed through several holes and it will breach the regime. Therefore, the possibility of appearance of this phenomena must be taken into account by design of machines of processing of thermoplastic materials.

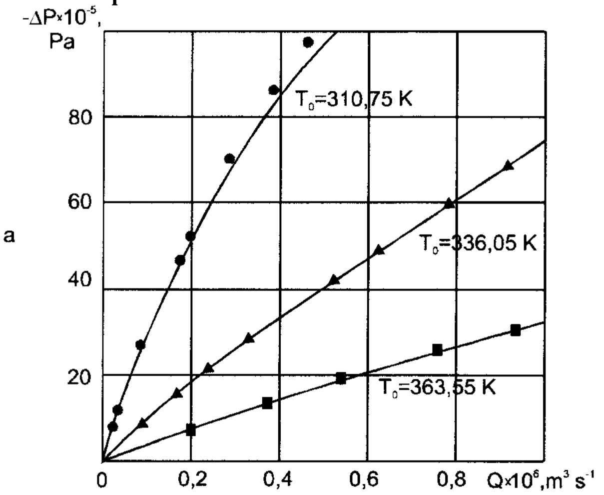

Figure 4. The comparison of the calculated results with the experimental data [11] for the flow Newtonian oil in the channel with $L= 4.54\cdot10^{-2}$m and $r_0= 1.31\cdot10^{-4}$ m. The lines are for calculation, the points for experiment.

To avoid this phenomena we can employ heat of die channels, negative feedback appears between characteristics of the flow and heat exchange with ambient, that leads to restoration of former regime.

Adequacy of preposition model of flow of polymer was checked by comparison of earlier published data with results obtained for conditions pointed at the articles. At the figure 4 it is shown data for flow of Newtonian mineral oil [11] with $\mu = 10^6 \rho \exp\left[\left(A/T^b\right) - 0.6\right]$, where $A = 1.372\cdot10^{10}, b = 3.81$. Comparison shows good agreement.

NOMENCLATURE

c- specific heat [J kg^{-1} K^{-1}], E- energy of activation of viscose flow [J mol^{-1}], K- heat exchange coefficient [J m^{-2} s^{-1}K^{-1}], P, P_0- pressure, current and inlet [N m^{-2}], W- flow rate [m^3 s^{-1}], r, r_0- radial co-ordinate [m], T, T_0- temperature of melt, current and inlet [K], V- velocity [m s^{-1}], x- longitudinal co-ordinate [m], Bi- Biot number, Gn- Nahme-Griffith number, Pe- Peclet number, Re- Reynolds number, $\frac{\rho V_0 r_0}{\mu_0}$, St_i- Stanton number for ith of layer, α- thermal diffusivity of melt [m^2 s^{-1}], λ- thermal conductivity of melt [J m^{-1} s^{-1}K^{-1}], μ- dynamic viscosity of melt [kg m^{-1} s^{-1}], ρ- density [kg m^{-3}].

Subscript: a- relating to ambient, i- number of layer, r, z- radial and longitudinal components of vector.

REFERENCES

1. Tadmor Z., Gogos C.G., *Principles of polymer processing,* p. 632, Mir, Moscow(1984).

2. Bostandzhiiyan S.A., Merzhanov A.G., Khudyaev S.I *Dokl. Akad. Nauk SSSR,*(Moscow), 1, 133(1965).

3. Benderskaya S.L., Khusid B.M., Shulman Z.P.*Izv. Akad. Nauk SSSR, Mekh. Zhid. I Gaza* (Moscow), 3, 3(1980).

4. Naidenov V.I., Brener A.M., Dilman V.V., Maksimov A.A. Contemporary problems of heat- and massexchange in chemical engineering.(Materials of international school-seminar). Ch. 3. (Minsk), 116(1987).

5. Naidenov V.I. *Teplofizika vysokikh temperatur* (Moscow), **24**, 1, 82(1986).

6. Naidenov V.I., Brener A.M., Dilman V.V. *Teor. Osn. Khim. Tekhnol.*, (Moscow) **23**, 3, 300 (1989).

7. Merzhanov A.G., Stolin A. M. *Zhurn. Prikl. Mechaniki i tekhn. fiziki.* (Novosibirsk), 1, 65(1974).

8. Malkin A.Ya., Chalykh A.E. Diffusion and viscosity of polymers. Methods of measurement. Moscow, "Khimiya". 1979.

9. Aleksanopolskii N.B., Naidenov V.I. *Teplofizika vysokikh temperatur* (Moscow), **17**, 4, 783(1979).

10. Shah Y.T., Pearson J.R.A. *Chem. Engng. Sci.* **29**, 6, 1485(1974).

11. Duda J.L., Klaus E.E., Lin S.C. Ind. Eng. Chem. Res. (Washington), **27**, 2, 352(1988).

12. Ponomarenko V.G., Potebnya G.F., Uliev L.M., Zhitinkin A.A., Olkhovikov O.A., *Inzh.-Fiz. Zh.*,(Minsk) **59**, 1, 158 (1990).

13. Ockendon H., *J. Fluid Mech.*, (London) **93**, 4, 737 (1979).

14. Uliev L.M., *Teor. Osn. Khim. Tekhnol.*, (Moscow) **29**, 3, 233 (1995).

15. Uliev L.M., *Teor. Osn. Khim. Tekhnol.*, (Moscow) **26**, 2, 213 (1992).